

MTH241 Fall 2024: Quiz 04

Name: Nomad
UID: Megalobox

Closed book, no calculator, show your work clearly.

1. (7pt) Determine a function f of two variables and a function g of one variable such that $F = g \circ f$ where $F(x, y) = \ln(\ln(x + y))$. What are the domains of f, g, F ? (Grading: **2pt**: working; **2pt**: f and g ; **3pt**: domains)

$$\begin{aligned}
 g(x) &= \ln(\ln(x)) \\
 f(x, y) &= x + y
 \end{aligned}
 \quad \Rightarrow \quad
 g \circ f(x, y) = \ln(\ln(x + y)) \quad \checkmark$$

$$\text{Domain of } g(x) = \ln(\ln(x)) = \{x : \ln(x) > 0\} = (1, \infty)$$

$$\begin{aligned}
 \text{Domain of } f(x, y) &= \mathbb{R}^2 \\
 \text{Domain of } \ln(\ln(x + y)) &= \{ (x, y) \in \mathbb{R}^2 : \ln(x + y) > 0 \} \\
 &= \{ (x, y) \in \mathbb{R}^2 : x + y > 1 \}
 \end{aligned}$$

2. (3pt) Use set notation to express the level curve of $f(x, y) = xy - \sin(xy)$ with respect to $c = 0$. (Grading: **1pt**: working; **2pt**: correct set notation)

$$\begin{aligned}
 \text{Level curve} &= \{ (x, y) : xy - \sin(xy) = 0 \} \\
 &= \{ (x, y) : xy = \sin(xy) \}
 \end{aligned}$$

Bonus: $z = \sin(z)$ is only true for $z = 0$

$$\text{Hence } xy = \sin(xy) \Leftrightarrow xy = 0$$

So the level set is $\{ (x, y) : xy = \sin(xy) \} = \{ (x, 0) \cup (0, y) : x, y \in \mathbb{R} \}$ i.e. the two axes

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